Date: 9/28/18

Chp: Chp. 2.1 → Rates of Change & Limits

Objectives:
- Calculate Avg/Instanteous Speed
- Definition of a Limit
- One & Two-Sided Limits

? • Sandwich Thrm

* Avg. Speed = $\frac{\text{distance covered}}{t}$ $D = rt$

$\frac{\Delta y}{\Delta x} = m =$ rate of change $\frac{D}{t} = r$

Ex. 1

A rock breaks loose from the top of a cliff. What is the avg. speed during the first 2 seconds of fall?

Free-fall eq → $y = 16t^2$   2 sec = t (2, 0)

$$\frac{\Delta y}{\Delta x} = \frac{16(2)^2 - 16(0)^2}{2-0} = \frac{64}{2} = 32 \text{ ft/sec}$$

* **Instantaneous Speed** = speed @ exact time.

**Ex. 2**

Find the instantaneous speed of the rock @ 2 sec. → 64 ft/sec

$$\frac{\Delta y}{\Delta x} = \frac{16(2)^2 - 16(1)^2}{2-1} = \frac{48}{1} = \cancel{24}\ 48 \text{ ft/sec}$$

$$\frac{16(2)^2 - 16(\cancel{1.75}\ 1.90)^2}{2-\cancel{1.75}\ 1.90} \qquad \frac{16(2)^2 - 16(1.5)^2}{2-1.5} = 56 \text{ ft/sec}$$

| t | ~~2~~ 1 | 1.5 | 1.75 | 1.90 | 1.99 | 1.999 |
|---|---|---|---|---|---|---|
| Avg speed | ~~24~~ 48 | 56 | 60 | 62.4 | 63.84 | 63.984 |

$$\frac{\Delta y}{\Delta x} = \frac{16(t+h)^2 - 16(t)^2}{h}$$

@exactly 2 secs.....

$$t$$

$$\frac{16(2+h)^2 - 16(\cancel{2})^{\cancel{2}\,4}}{h}$$

$$\frac{16(4+4h+h^2)-64}{h}$$

$$\frac{\cancel{64}+64h+16h^2-\cancel{64}}{h}$$

$$\frac{64\cancel{h}+16\cancel{h}^{2\,h}}{\cancel{h}} = 64+16\overset{0}{h}$$

* Instantaneous speed is the limit of the avg. speed function as the elapsed time approaches 0.

* <u>Limit</u> = Let $f(x)$ be a function defined on an interval that contains $x=c$ except possibly @ $x=c$.

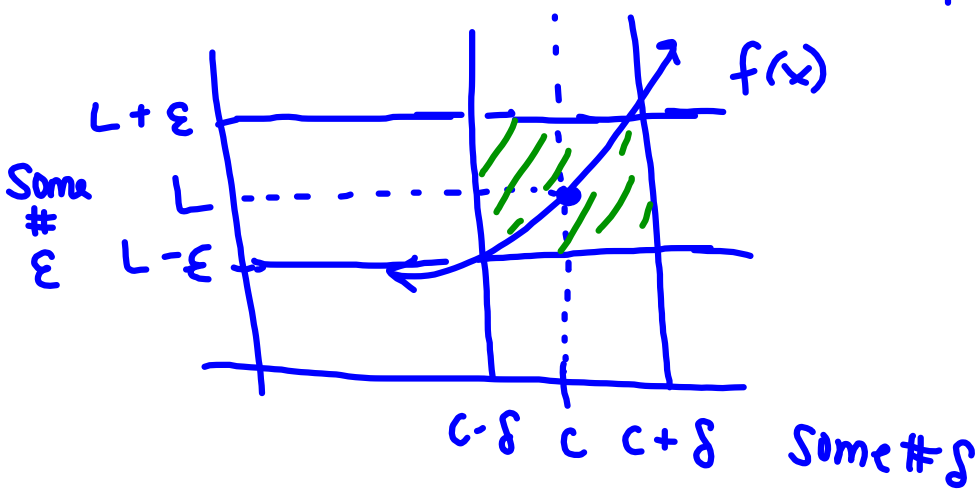
$$\lim_{x \to c} f(x) = L$$

epsilon

if for every number $\varepsilon > 0$ there is some # $\delta > 0$ such that:

delta

$$|f(x) - L| < \varepsilon \text{ whenever } 0 < |x - c| < \delta$$




Ex: $\lim_{\substack{h \to 0 \\ x \to c}} \overset{f(x)}{64+16h} = \overset{L}{64}$

Ex. 1

Find the $\lim_{x \to 0} \frac{\sin x}{x} = 1$

Ex. 2

$\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = 2$

Ex. 3

$\lim_{x \to 1} \begin{cases} \frac{x^2-1}{x-1} & x \neq 1 \\ 1 & x = 1 \end{cases} = 2$

Ex. 4

$\lim_{x \to 1} x + 1 = 2$

## Thrm 1: Properties of Limits

If $L$, $M$, $c$, & $k$ are $\mathbb{R}$ #s and

if $\lim_{x \to c} f(x) = L$ & $\lim_{x \to c} g(x) = M$:

1) Sum Rule

$$\lim_{x \to c} (f(x) + g(x)) = L + M$$

2) Difference Rule

$$\lim_{x \to c} (f(x) - g(x)) = L - M$$

3) Product Rule

$$\lim_{x \to c} (f(x) \cdot g(x)) = L \cdot M$$

4) Constant Multiple Rule

$$\lim_{x \to c} (k \cdot f(x)) = k \cdot L$$

5) Quotient Rule

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{M}, \quad M \neq 0$$

6) Power Rule

If $r$ & $s$ are integers & $s \neq 0$,

$$\lim_{x \to c} (f(x)^{r/s}) = L^{r/s}$$

Ex. 5

$$\lim_{x \to c} (x^3 + 4x^2 - 3) = c^3 + 4c^2 - 3$$

Ex. 6

$$\lim_{x \to c} \left( \frac{x^4 + x^2 - 1}{x^2 + 5} \right) = \frac{c^4 + c^2 - 1}{c^2 + 5}$$

## Thrm 2: Polynomial & Rational Functions

1) If $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_0$

$a_5 x^5 + a_4 x^4 + a_3 x^3 \quad + c$

then

$$\lim_{x \to c} f(x) = f(c)$$

2) If $f(x)$ & $g(x)$ are polynomials then

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{f(c)}{g(c)}, \quad g(c) \neq 0$$

Ex. 7

Find $\lim_{x \to 3} \left[x^2(2-x)\right] = -9$

Ex. 8

$$\lim_{x \to 2} \frac{x^2+2x+4}{x+2} = \frac{2^2+2(2)+4}{2+2} = 3$$

Ex. 9

$$\lim_{x \to 0} \frac{\tan x}{x} = 1$$

$$\lim_{x \to 0} \frac{\sin x}{x \cos x} = \frac{\sin x}{x} \cdot \frac{1}{\cos x}$$

$$\lim_{x \to 0} \frac{\sin x}{x} \cdot \lim_{x \to 0} \frac{1}{\cos x}$$

$$1 \cdot \frac{1}{1} = 1$$

Ex. 10

$$\lim_{x \to 2} \frac{x^3-1}{x-2} = \text{DNE}$$

# One-Sided and Two-Sided Limits

Sometimes the value of f tends to different limits as $x \to c$ from opposite sides.

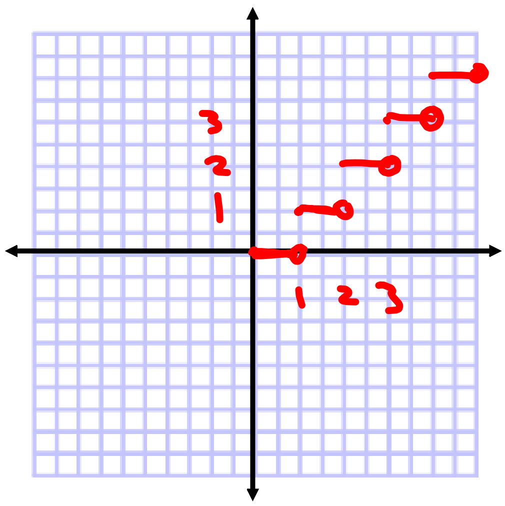
Right-hand $\lim_{x \to c^+} (f(x))$

Left-hand $\lim_{x \to c^-} f(x)$

$\boxed{\text{Ex 10}}$

$y = [\![ x ]\!]$

$y = \text{int}(x)$




$\lim_{x \to 3^-} f(x) = 2$

$\lim_{x \to 3^+} f(x) = 3$

## Theorem 3

A function $f(x)$ has a limit as $x \to c$ if and only if the right-hand and left-hand limits exist and are equal.

$\boxed{\text{Ex 11}}$

$$f(x) = \begin{cases} -x+1 & 0 \le x < 1 \\ 1 & 1 \le x < 2 \\ 2 & x = 2 \\ x-1 & 2 < x \le 3 \\ -x+5 & 3 < x < 4 \end{cases}$$

a. $\lim_{x \to 0^+} f(x) = 0+1 = 1$

b. $\lim_{x \to 0^-} f(x) = \text{DNE}$

$\lim_{x \to 0} f(x)$ DNE

c. $\lim_{x \to 1^+} f(x) = 1$

d. $\lim_{x \to 1^-} f(x) = -1+1 = 0$

$\lim_{x \to 1} f(x)$ DNE

e. $\lim_{x \to 2^+} f(x) = 2-1 = 1$

f. $\lim_{x \to 2^-} f(x) = 1$

$\lim_{x \to 2} f(x) = 1$

g. $\lim_{x \to 3^+} f(x) =$

h. $\lim_{x \to 3^-} f(x) =$

$\lim_{x \to 3} f(x)$

# Homework:

p. 66-67

#1-6, 7-27 odds, 31-37 odds, 38, 39-43 odds, 51